Acceleration

Key Ideas

- Acceleration is the rate of change of velocity with respect to time.
- The average acceleration of an object during a time period is the change in the velocity of the object divided by the length of the time interval.
- The acceleration of an object, sometimes called the instantaneous acceleration, is the time derivative of the velocity, $\vec{v}(t)$. This is equal to the average acceleration in the limit as the length of the time interval goes to zero.
- The acceleration is a vector with its direction determined by the change in the velocity.

Learning Objectives

By the end of this section, you should be able to:

- distinguish between the acceleration at a specific time (the instantaneous acceleration) and the average acceleration,
- determine or estimate the acceleration of an object from a graph of velocity vs. time,
- determine the direction of the acceleration, or the sign of the acceleration in a one-dimensional coordinate system, from a graph of position vs. time, and
- determine the acceleration of an object from the derivative of the velocity, $d\vec{v}(t)/dt$.

Our next step in understanding the motion of objects is to describe how velocity changes and define **acceleration**, the rate at which the velocity changes.

We need to be careful about the term "accelerate", as this is used somewhat differently in everyday conversation from how it is used in physics. For example, when driving a car, the driver steps on the "accelerator" pedal to increase speed and steps on the brake pedal to slow down, making a distinction between the two changes in velocity in common usage. In physics, however, acceleration refers to ***any*** change in velocity, whether it is an object "speeding up," "slowing down," or "changing direction" with or without a change in speed. Confusion between velocity and acceleration is common. It is very important to fully grasp the distinction between velocity and its rate of change.

Average Acceleration

The definition of the average acceleration of an object is similar to other examples of a "rate of change," such as the rate of change of position is the velocity. We now define average acceleration as the rate of change of the velocity of an object.

AVERAGE ACCELERATION

Average acceleration is the rate at which velocity changes during some time interval, from t_i to t_f. The average acceleration is a vector that points in the direction of the *change* in velocity, $\Delta\vec{v}$. This can be a change in magnitude or direction of the velocity, and can be in any direction relative to the velocity itself.

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_f) - \vec{v}(t_i)}{t_f - t_i} \qquad (3.16)$$

For motion in one dimension, with a defined coordinate system, the x-component of the acceleration is

$$\text{One Dimension:} \qquad a_{x,ave} = \frac{\Delta v_x}{\Delta t} = \frac{v_x(t_f) - v_x(t_i)}{t_f - t_i i} \qquad (3.17)$$

The SI units for acceleration are m/s^2.

Using SI units, acceleration is the change in velocity, with units meters per second, divided by the change in time, in units of seconds. This gives the units of acceleration:

$$\text{Meters per second divided by seconds: } \frac{m/s}{s} = \frac{m}{s^2}$$

This means the acceleration indicates how many meters per second the velocity changes every second. For example, suppose a car starting from rest has an acceleration of $2\ m/s^2$. After one second, its speed is $2\ m/s$. Two seconds, $4\ m/s$, three seconds, $6\ m/s$, etc., until the driver chooses to stop accelerating to maintain a constant velocity.

A change in velocity can be a change in the magnitude of the velocity (or speed) and/or a change in direction. For example, consider a runner initially traveling at $10\ km/h$, due east when she reverses direction and continues her run at $10\ km/h$ due west. Her initial and final ***speed*** is the same, but her initial and final velocities are not. Thus, her average acceleration is not zero because the change in velocity is not zero. Δv_x is towards the west.

Because the acceleration is related to the ***change in velocity*** it can be in any direction relative to the velocity. The direction of the acceleration relative to the direction of the velocity is important for understanding the physics of a problem. Consider the motion of an object in one dimension and its average acceleration over a short time interval. If an object speeds up its acceleration is in the same direction as the velocity and if the object slows down its acceleration is opposite to the direction of its velocity.

Note: "Slowing down" is often referred to as *deceleration* in everyday conversation. We will not use this term because it often leads to errors in direction, or sign errors when using a coordinate system.

Figure 3.12 A subway train in Sao Paulo, Brazil, slows down as it enters a station. This implies the train accelerates in the direction opposite to the velocity. (credit: modification of work by Yusuke Kawasaki)

In the case of the train in Figure 3.12, the acceleration is in the direction opposite to the direction of the velocity as the train slows to a stop in the station. If we choose a coordinate system with the initial velocity in the positive x-direction, the x-component of the acceleration is negative. Of course, if we use a coordinate system where the initial velocity is in the negative x-direction, the acceleration will be in the positive x-direction. In both coordinate systems, the physics is the same, and the train slows down.

As another example, consider the two situations shown in Figure 3.13. In both cases, a and b, an object has an initial velocity $\vec{v}_i$ (top) and at a later time the velocity $\vec{v}_f$ is in the opposite direction (bottom). This indicates the object turned around in both cases. Using the coordinate system shown with the positive x-direction to the right, we can use the x-components of the initial and final velocities to determine the x-component of the average acceleration.

a. Particle (a): $v_i > 0,\ v_f < 0,\ a_{ave} = \frac{v_f - v_i}{\Delta t} < 0$. The average acceleration is to the left, the x-component is negative.
b. Particle (b): $v_i < 0,\ v_f > 0,\ a_{ave} = \frac{v_f - v_i}{\Delta t} > 0$. The average acceleration is to the right, the x-component is positive.

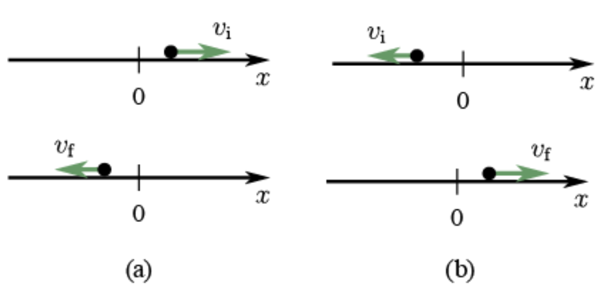

Figure 3.13 Initial and final velocities during a time interval for two objects are shown. For object (a), the x-component of the velocity goes from the positive to negative x-direction (right to left) giving an average acceleration in the negative x-direction. For object (b) the x-component of the velocity goes from

negative to positive (left to right) giving an average acceleration in the positive x-direction.

Both objects slow down, turn around, and speed up again in the other direction. Their motions are very similar. The result that the x-component of the average velocity is negative in the first case and positive in the second case is a result of the choice of coordinate system.